

The masses of vector mesons in holographic QCD at finite chiral chemical potential

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Abstract

Central heavy-ion collisions may induce sizeable fluctuations of the topological charge. This effect is expected to distort the dispersion relation for the hadron masses. We construct a general setup for a compact description of this phenomenon in the framework of bottom-up holographic approach to QCD. A couple of soft wall holographic models are proposed for the vector mesons. The states having different circular polarizations are shown to have different effective mass. The requirement of stability imposes strong constraints on the possible choice of models.

1 Introduction

Recently the study of light mesons by means of a bottom-up holographic approach has attracted a lot of attention (see, e.g., the short surveys [1]). Usually the spectra of resonances and the related physics are analysed at vacuum conditions. However, in view of current experiments with relativistic heavy ion collisions at RHIC, GSI, and CERN, it is useful to include in the holographic models some non-trivial external conditions. One of theoretical methods consists in incorporation of the Chern-Simons (CS) term in a holographic action. This permits to address holographically such problems as the magnetic susceptibility of the quark condensate [2], the chiral magnetic effect [3, 4], some subtle questions in the behavior of the correlation functions [5, 6], and derivation of the $O(p^6)$ Chiral Perturbation Theory low-energy constants [7].

It has been demonstrated recently [8] that a suitable CS term may lead to interesting meson phenomenology. This CS term is motivated by the possibility of local parity breaking taking place in baryonic matter; a phenomenon that could be triggered by large topological fluctuations taking place in central heavy ion collisions. For light quarks in a quasi-equilibrium situation the

creation of a topological charge translates immediately into the generation of a finite chiral chemical potential

Inspired by these observations, in the present paper we analyse the impact of the CS term in the background considered in Ref. [8] on the mass spectrum of the vector mesons in the Soft Wall (SW) holographic model [9]. This will give a generalization of results of Ref. [8] to higher radial excitations in the large- N_c limit of QCD. Our choice of the model is motivated by its nice property of possessing the linear Regge like spectrum which is expected in the first approximation both experimentally [10] and in the string like models of hadrons. To make the consideration clear we will restrict ourselves by the simplest version of the SW model.

The paper is organized as follows. The general holographic setup is presented in Section 2. The analysis of impact of the axial chemical potential on the mass spectrum is given in Section 3. A couple of exactly solvable models are constructed in Section 4. A short discussion of our results are contained in the concluding Section 5.

2 The holographic setup

We consider the gauge SW model [9] with 5D Abelian fields L and R dual (on the AdS_5 boundary) to the sources of the left and right 4D vector currents. A parity-odd 5D CS term is added to the action,

$$S = S_{\text{free}}[L] + S_{\text{free}}[R] + S_{\text{CS}}[L] - S_{\text{CS}}[R], \quad (1)$$

$$S_{\text{free}}[B] = -\frac{1}{8g_5^2} \int d^4x dz e^\varphi \sqrt{g} B_{MN} B^{MN}, \quad B = L, R, \quad (2)$$

$$S_{\text{CS}}[B] = -k \int d^4x dz \epsilon^{MNABC} B_M B_N A_B B_C. \quad (3)$$

The 5D space is the AdS_5 one with the metric

$$ds^2 = \frac{R^2}{z^2} (dx_\mu dx^\mu - dz^2), \quad \mu = 0, 1, 2, 3, \quad (4)$$

where R is the radius of the AdS_5 space and $z > 0$ represents the holographic coordinate. The dilaton background e^φ is not yet fixed for generality. As usual, the constants g_5 and k are fixed by matching to the ultraviolet asymptotics of the two-point vector correlator [11] and to the axial anomaly, respectively

$$\frac{g_5^2}{R} = \frac{12\pi^2}{N_c}, \quad k = \frac{N_c}{24\pi^2}. \quad (5)$$

In terms of the vector, $V = (L + R)/2$, and axial-vector, $A = (R - L)/2$, fields the free and CS parts of the action can be rewritten as (Lorentz indices are lowered)

$$S_{\text{free}} = -\frac{1}{4g_5^2} \int d^4x dz \frac{e^\varphi}{z} (V_{MN}^2 + A_{MN}^2), \quad (6)$$

$$S_{\text{CS}} = -k \int d^4x dz \epsilon^{MNABC} A_M (V_{NA} V_{BC} + A_{NA} A_{BC}). \quad (7)$$

If one wishes to provide conservation of the 4D vector current, the Bardeen surface counterterm must be added,

$$S_{\text{B}} = 2k \int d^4x \epsilon^{\mu\nu\lambda\rho} A_\mu V_\nu V_{\lambda\rho}. \quad (8)$$

Then one obtains the standard result for the covariant anomaly [12],

$$\partial_\mu J_\mu^V = 0, \quad \partial_\mu J_\mu^A = 3k V_{\mu\nu} \tilde{F}_{\mu\nu} + k A_{\mu\nu} \tilde{A}_{\mu\nu}, \quad (9)$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}$.

In contrast to the finite density effects, the thermal corrections to the meson masses appear in the next-to-leading order in the large- N_c counting as they emerge due to the pion loops. On the other hand, the holographic approach is inherently large- N_c one. For this reason we will not include the finite temperature effects into our considerations.

3 Embedding the axial chemical potential

In Ref. [8] it was assumed that the axial chemical potential μ_5 arises from a time-dependent but spatially homogeneous background of a pseudoscalar field $a(t)$ such that $\mu_5 = \dot{a}(t)$. In the 5D setup, we will treat A_M as a background axial-vector field and relate $a(t)$ to the z -component of A_M . Namely, we assume for the vacuum expectation value of the vector field that

$$\langle A_M \rangle = \langle A_z \rangle = \mu_5 x_0 f(z), \quad (10)$$

where the shape function $f(z)$ will be specified later. The expressions (9) and (10) lead to the following form for the vector part of the action (1),

$$S = \frac{R}{g_5^2} \int d^4x dz \left(-\frac{e^\varphi}{4z} V_{MN}^2 + \xi \mu_5 f \epsilon^{05ABC} V_A \partial_B V_C \right). \quad (11)$$

Here $\xi = \frac{2kg_5^2}{R} = 1$ (see (5)).

In the axial gauge $V_z = 0$, the equation of motion reads

$$\partial_z \left(\frac{e^\varphi}{z} \partial_z V_\mu \right) - \frac{e^\varphi}{z} \partial_\mu^2 V_\mu - 2\mu_5 f \epsilon_{mik} \partial_i V_k = 0. \quad (12)$$

The small latin indices denote the usual space coordinates, $m, i, k = 1, 2, 3$. Making the 4D Fourier transform, $V_\mu(x, z) = \int d^4 p e^{ipx} V_\mu(p, z)$ and assuming the standard plane wave ansatz, $V_\mu(p, z) = \varepsilon_\mu v(z)$, we arrive at the equation for the particle-like excitations,

$$\left[\partial_z \left(\frac{e^\varphi}{z} \partial_z v \right) + \frac{e^\varphi}{z} p^2 v \right] \varepsilon_\mu + i2\mu_5 f \epsilon_{mik} p_i \varepsilon_k v = 0. \quad (13)$$

The physical spectrum is given by the eigenvalues $p_n^2 = m_n^2$ of normalizable solutions of Eq. (13). However, the last term induces the mixing between different polarizations. We must find a basis diagonalizing the Eq. (13).

For convenience, let us introduce the notation for the differential operator

$$\hat{F} = \partial_z \left(\frac{e^\varphi}{z} \partial_z \right) + \frac{e^\varphi}{z} p^2. \quad (14)$$

The space-like part of Eq. (13) can be rewritten in the vector form

$$\left(\hat{F} \vec{\varepsilon} + i2\mu_5 f \vec{p} \times \vec{\varepsilon} \right) v = 0. \quad (15)$$

The equation (15) is diagonalized with the help of the projectors,

$$\hat{P}_{ik}^\parallel = \frac{p_i p_k}{p^2}, \quad (16)$$

$$\hat{P}_{ik}^\pm = \frac{1}{2} \left[\delta_{ik} - \frac{p_i p_k}{p^2} \pm \frac{i}{|\vec{p}|} \epsilon_{ikn} p_n \right]. \quad (17)$$

The projectors (17) on the "circular" polarizations have the following evident properties,

$$\hat{P}^\pm \hat{P}^\mp = 0, \quad \hat{P}^\pm \hat{P}^\pm = \hat{P}^\pm, \quad \text{tr} \hat{P}^\pm = 1, \quad \hat{P}^\pm \vec{p} = 0, \quad \hat{P}^+ + \hat{P}^- = \hat{P}^\perp. \quad (18)$$

Now we change the basis for the space-like polarizations from $\vec{\varepsilon} = (\varepsilon_1, \varepsilon_2, \varepsilon_3)$ to $(\varepsilon_\parallel, \varepsilon_-, \varepsilon_+)$. The equation (15) takes the form

$$\hat{F} v^\parallel = 0, \quad (19)$$

$$\left(\hat{F} \pm 2\mu_5 f |\vec{p}| \right) v^\pm = 0. \quad (20)$$

Thus, the longitudinal and circular polarizations will have different masses, with the latter being dependent on the momentum. In other words, instead of a "peak" corresponding to a given meson we will see three "peaks" with the splitting depending on the value of three-dimensional momentum. A similar phenomenon in a different situation was obtained in Refs. [8, 13].

4 Solvable models

We have not yet specified the dilaton background $\varphi(z)$ and the shape function $f(z)$ for the axial chemical potential. Various holographic models for the "splitting phenomenon" can be obtained by fixing these functions. The simplest solvable SW model resulting in a Regge-like spectrum is given by the background [9].

$$\varphi = -\lambda^2 z^2. \quad (21)$$

Let us accept this background and consider the longitudinal polarization. Following Ref. [9], we use the substitution

$$v(z) = e^{\lambda^2 z^2/2} \sqrt{z} \psi(z), \quad (22)$$

to convert the Eq. (19) into the Schrödinger form

$$-\partial_z^2 \psi_n^\parallel + \left(\lambda^4 z^2 + \frac{3}{4z^2} \right) \psi_n^\parallel = m_n^2 \psi_n^\parallel. \quad (23)$$

Introducing the dimensionless variable $y = \lambda z$, Eq. (23) for the discrete mass spectrum $m_n^2 = p_n^2$ of the longitudinal (and time like) polarization transforms into

$$-\partial_y^2 \psi_n^\parallel + \left(y^2 + \frac{3}{4y^2} \right) \psi_n^\parallel = \frac{m_{n,\parallel}^2}{\lambda^2} \psi_n^\parallel. \quad (24)$$

The normalized solutions are ($n = 0, 1, 2, \dots$)

$$\psi_n^\parallel = \sqrt{\frac{2n!}{(1+n)!}} e^{-y^2/2} y^{3/2} L_n^1(y^2), \quad (25)$$

where L_n^1 denote the associated Laguerre polynomials. The mass spectrum is given by the corresponding eigenvalues,

$$m_{n,\parallel}^2 = 4\lambda^2(n+1), \quad (26)$$

where the parameter λ controls the slope of the radial Regge trajectory.

To obtain the spectrum of circular polarizations we need to specify the function $f(z)$. The choice of $f(z)$ must comply with the requirement of correct UV/IR behavior of the action with respect to conformal symmetry. This constraint leaves, however, much freedom. We will be interested in the exactly solvable cases. A simple possibility of this sort is given by the ansatz

$$f = \frac{b}{2} \frac{e^{-\lambda^2 z^2}}{z}. \quad (27)$$

Here b is a dimensionless constant. After the substitution (22) the Schrödinger equation (24) acquires an additional contribution,

$$-\partial_y^2 \psi_n^\pm + \left(y^2 + \frac{3}{4y^2} \pm \frac{b\mu_5}{\lambda^2} |\vec{p}| \right) \psi_n^\pm = \frac{m_{n,\pm}^2}{\lambda^2} \psi_n^\pm. \quad (28)$$

The eigenfunctions remain the same as in (25) but the mass spectrum is shifted,

$$m_{n,\pm}^2 = 4\lambda^2(n+1) \pm b\mu_5 |\vec{p}|. \quad (29)$$

Thus, the massive vector fields split into three polarizations with masses $m_{n,-} < m_{n,\parallel} < m_{n,+}$. The mass splitting is linearly dependent on the value of the spatial momentum \vec{p} . The formula (29) can be considered as a generalization of the result of Ref. [8] to the radially excited spectrum.

The matching of (29) with the corresponding expression in Ref. [8],

$$m_{V,\pm}^2 = m_V^2 \pm \zeta |\vec{p}|, \quad (30)$$

allows to estimate the constant b . Within the effective model studied in Ref. [8], $\zeta = N_c g_\rho^2 \mu_5 / 8\pi^2$, where g_ρ is related with the mass of vector particle, $m_\rho^2 = 2g_\rho^2 f_\pi^2 \simeq m_\omega^2$. Substituting the phenomenological values for m_ρ and for the weak pion decay constant f_π , one obtains the estimate $\zeta \simeq 1.5\mu_5$. We remark that this estimate can be made without phenomenological values if one uses the formula for m_ρ from the QCD sum rules in the large- N_c limit (see, e.g., [14]), $m_\rho^2 = 24\pi^2 f_\pi^2 / N_c$ that yields $g_\rho = 12\pi^2 / N_c$ and leads directly to $\zeta = \frac{3}{2}\mu_5$. Comparing (29) and (30) we arrive at the value $b = \frac{3}{2}$.

Using the results of Ref. [15], the background (21) can be generalized to

$$\varphi = -\lambda^2 z^2 \log U^2(c, 0; \lambda^2 z^2), \quad (31)$$

where U is the Tricomi hypergeometric function. Then the dimensionless parameter c in (31) will control the shift of the radial Regge trajectories: $n+1$ in (26) and (29) will be replaced by $n+1+c$.

The model was designed such that the relation $m_{n,+}^2 - m_{n,-}^2 = \text{const}$ holds for any n . Other possibilities are of course possible. For instance, one can construct a solvable model with the relation $m_{n,+}^2 / m_{n,-}^2 = \text{const}$. This is achieved via replacing (27) by

$$f = \frac{\tilde{b}}{2} z e^{-\lambda^2 z^2}. \quad (32)$$

Introducing the variable

$$y = \left(1 \pm \frac{\tilde{b}\mu_5}{\lambda^4} |\vec{p}| \right)^{\frac{1}{4}} \lambda z, \quad (33)$$

the analogue of Eq. (28) looks as follows

$$-\partial_y^2 \psi_n^\pm + \left(y^2 + \frac{3}{4y^2}\right) \psi_n^\pm = \frac{m_{n,\pm}^2}{\lambda^2 \sqrt{1 \pm \frac{\tilde{b}\mu_5}{\lambda^4} |\vec{p}|}} \psi_n^\pm, \quad (34)$$

resulting in the mass spectrum

$$m_{n,\pm}^2 = 4\lambda^2 \sqrt{1 \pm \frac{\tilde{b}\mu_5}{\lambda^4} |\vec{p}|} (n+1). \quad (35)$$

The eigenfunctions (25) are now momentum- and μ_5 -dependent because the variable y (33) depends on $|\vec{p}|$ and μ_5 . The matching to Eq. (30) for $n=0$ for small $|\vec{p}|$ gives $\tilde{b} = \frac{3}{4}\lambda^2$. In this scenario, the contribution of μ_5 to the masses grows with n .

Since the effective masses of polarized modes depend on the spatial momentum in the models under consideration, a certain care must be exercised to provide stability and, as a consequence, to avoid the superluminal propagation. From (29) we have for the energy

$$p_{0,n}^2 = \vec{p}^2 + 4\lambda^2(n+1) \pm b\mu_5 |\vec{p}|. \quad (36)$$

In this respect, the ε_- polarization is dangerous. Imposing the condition $p_{0,n}^2 \geq 0$ we arrive at

$$\mu_5^2 \leq \frac{16\lambda^2}{b} (n+1). \quad (37)$$

The strongest limitation comes from the ground state $n=0$,

$$\mu_5 \leq \frac{4\lambda}{b}. \quad (38)$$

Consider now the group velocity $v = \frac{dp_0}{d|\vec{p}|}$. It cannot exceed the speed of light in the vacuum, $v \leq 1$. For (36) this yields

$$\frac{2|\vec{p}| \pm b\mu_5}{2\sqrt{\vec{p}^2 + 4\lambda^2(n+1) \pm b\mu_5 |\vec{p}|}} \leq 1. \quad (39)$$

The constraint (39) leads to the same limitations (37) and (38).

The analogue of constraint (39) for the second model considered above takes the form

$$\frac{|\vec{p}| \pm \frac{b\mu_5(n+1)}{\lambda^2 \sqrt{1 \pm \tilde{b}\mu_5 |\vec{p}|/\lambda^4}}}{\sqrt{\vec{p}^2 + 4\lambda^2 \sqrt{1 \pm \tilde{b}\mu_5 |\vec{p}|/\lambda^4} (n+1)}} \leq 1. \quad (40)$$

It is seen that the condition (40) cannot be fulfilled at any n — the larger is n the stronger is limitation on μ_5 . Since n labels the Kaluza–Klein modes of a five-dimensional field, all infinite number of resonances must be present simultaneously in agreement with the limit $N_c \rightarrow \infty$. Thus, the second solvable model should be disregarded.

5 Concluding discussions

According to some estimates of Ref. [8], the natural value of μ_5 in the central heavy ion collisions is in the few hundreds of MeV. Such values are far below the limitation (38),

$$\mu_5 \leq \frac{4\lambda}{b} \approx \frac{8\lambda}{3} \approx 1400 \text{ MeV}, \quad (41)$$

where the value $\lambda \approx 530$ MeV is taken from the typical slopes of the vector radial trajectories. Thus, the first model passes the stability criterium. We have shown that this criterium imposes a strong constraint on the possible choice of models. In particular, it seems to falsify the second model constructed in the previous Section.

In principle, the effect of mass splitting between different polarizations can be quite large. The masses of ε_+ and ε_- circular polarizations of neighboring resonances can even overlap.

We expect that the contribution of the axial chemical potential to the hadron masses is approximately constant, i.e. it does not depend on the radial number n . In this regard, the first constructed model looks physical.

The splitting of effective masses of two polarizations signifies the parity breaking in the medium. In the framework of Ref. [8], this effect is related with the violation of 4D Lorentz invariance due to the time-dependent background which is present in the Lagrangian from the very beginning. In the presented holographic description, we have essentially the same situation.

The experimental implications of the presence of the axial chemical potential are discussed in detail in Ref. [16].

Finally we mention some possible future directions. First of all, it would be interesting to study the impact of the axial chemical potential on the masses of vector mesons in the top-down holographic approach. Second, it makes sense to include a non-zero isospin chemical potential μ_I into our consideration. In the low-energy QCD, μ_I contributes to the masses of charged pions, $m_{\pi^\pm} \approx m_{\pi^0} \pm \mu_I$, and triggers the pion condensation at values $\mu_I > m_{\pi^0}$ [17]. This result is reproduced by the hard-wall bottom-up holographic model [18] and by the Sakai-Sugimoto top-down holographic model [19]. In addition, the latter analysis shows that at $\mu_I \gtrsim 1.7m_\rho$, the

lowest vector meson also condenses. It is thus interesting to verify this effect in the bottom-up approach and then to study the interplay of μ_I and the axial chemical potential.

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